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Growth, Residential Distribution, and Land Price in an Integrated Solow's Growth and Alonso's Residential Model

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Abstract

This study builds a spatial growth model with residential distribution. The model is a synthesis of the Solow growth and Alonso residential models with endogenous land value. A unique feature of the study is that it deals with dynamic interactions between land value, land rent, wealth accumulation, amenity change, and residential location. It models the economic production and growth mechanism on the basis of the Solow growth model and the residential distribution on the basis of the Alonso model. The determination of the land value is based on Zhang's recent work on economic growth with endogenous land value. We simulate the motion of the economy over time and space. We carry out comparative dynamic analysis with regard to the rate of interest, the total productivity, and the propensity to save.

Keywords: *Economic growth, Travel time, Land rent, Land value, Residential location.*

1. Introduction

This study builds a spatial growth model with residential distribution. The model is a synthesis of the Solow growth and Alonso residential models for a small-open economy with endogenous land value. A unique feature of the study is that it deals with dynamic interactions between determine land value, land rent, wealth accumulation, amenity and change, residential distribution. It models the economic production and growth mechanism on the basis of the Solow growth model and the residential distribution on the basis of the Alonso model (Solow, 1956; Alonso, 1964). The integration of the Solow and Alonso is carried out by Zhang (2010). The description of a small-open growth economy is based on the literature of open growth economies (e.g., Obstfeld and Rogoff, 1996). The determination of the land value is based on Zhang's recent work on economic growth with endogenous land value (Zhang, 2014). The paper integrates different economic mechanisms into an integrated framework. We simulate the motion of the economy over time and space. We carry out comparative dynamic analysis with regard to the rate of interest, the total productivity, and the propensity to save. The paper is based on an unpublished presentation presented in an international conference by Zhang (2018). Section 2 defines the spatial growth model with endogenous land value and residential location for a small-open economy. Section 3 simulates the motion of the economic system. Section 4 carries out comparative dynamic analysis with regard to the rate of interest, the total productivity of the production sector, and the propensity to save. Section 5 concludes the study. The appendix proves the main results in section 3.

2. The Model

We now build the model of dynamic interdependence between economic growth and residential density change over space for a small-open economy. The model is a synthesis of the basic features of the Solow growth model and the Alonso residential model with Zhang's approach to household behavior. The modelling of the residential land-use follows the Alonso model. The economic system is an open urban economy built on a flat featureless plain. All workers reside over the city and work in the CBD (central business district). People travel only between dwelling sites and the CBD. An individual reside only at one location. The only spatial characteristic that directly matters is the distance from the residential site to the CBD. The isolated state consists of a finite strip of land extending from a fixed central business district with constant unit width. The system is geographically linear and consists of two parts - the CBD and the residential area. We use L to stand for the fixed (territory) length of the state. We assume that all economic activities are concentrated in the CBD. We use ω to represent the distance from the CBD to a point in the residential area. We use $R(t, \omega)$ and $p(t, \omega)$ to represent the land rent and land price at location ω at time t. The households occupy the residential area. We assume that the industrial product can be either invested or consumed. Housing is measured by lot size. The total labor force is fully employed by the industrial sector. We select industrial good to serve as numeraire. As we assume that the transportation cost of workers to the city is dependent on the travel distance, land rent for housing should be spatially different. We use $K(t)$ to stand for the total capital stock.

2.1. The Total Labor Input

We use $n(t, \omega)$ to denote the residential density at ω. We assume that all the workers work the same time, irrespective of where they live. The population N is homogenous and constant. The width of the urban area is assumed to be unity. According to the definition of $n(t, \omega)$, we have

$$N = \int_0^L n(t, \omega) d\omega. \tag{1}$$

2.2. The Production Sector

Let $F(t)$ stand for the production function. The production function is specified as follows

$$F(t) = AK^{\alpha}(t)N^{\beta}, \quad \alpha, \beta > 0, \quad \alpha + \beta = 1, \tag{2}$$

where A, α, and β are positive parameters. The rate of interest r^* and wage rate $w(t)$, are determined by markets. We assume that r^* is constant and determined by international markets. The marginal conditions are given by

$$r^* + \delta_k = \frac{\alpha F(t)}{K(t)}, \quad w(t) = \frac{\beta F(t)}{N(t)}, \tag{3}$$

where δ_k is the depreciation rate of physical capital. From (2) and (3) we solve

$$K(r^*) = \left(\frac{\alpha A}{r^* + \delta_k}\right)^{1/\beta} N, \quad w(r^*) = \frac{\beta A K^{\alpha}}{N^{\alpha}}. \tag{4}$$

2.3. The Relation between the Lot Size and Residential Density

We assume that all housing is residential. Let us denote $l(t, \omega)$ the lot size of the household at ω. According to the definitions of l and n, we have

$$n(t, \omega) = \frac{1}{l(t, \omega)}, \quad 0 \le \omega \le L. \tag{5}$$

2.4. Choice between Physical Wealth and Land

Consider now a household with one unity of money. He can either invest in capital good thereby earning a profit equal to the net own-rate of return r^* or invest in land thereby earning a profit equal to the net own-rate of return $R(t, \omega)/ p(t, \omega)$. The two options are assumed to yield equal returns, i.e.

$$\frac{R(t, \omega)}{p(t, \omega)} = r^*. \tag{6}$$

2.5. Travel Time and Cost to the CBD

A resident decides the time distribution between leisure time and travel time. Let T_0 and $\Gamma(\omega)$ respectively stand for the total available time and the time spent on traveling between the residence and CBD. We have

$$T(\omega) + \Gamma(\omega) = T_0, \tag{7}$$

where $T(\omega)$ is the leisure time that the household at ω enjoys. This study assumes that the travel cost $c_T(\omega, t)$ from location ω to the CBD is dependent on the distance as follows

$$c_T(t, \omega) = \bar{c}(t) + c_0(\omega). \tag{8}$$

2.6. Land Ownership, Current Income, and Disposable Income

Let $\bar{k}(\omega, t)$ stand for the representative household's physical wealth, and $a(\omega, t)$ for the value of land owned by the household at location ω. The total value of land owned by the household at ω is the sum of all the value of land the household owns in the economy. We have.

$$a(t, \omega) = \int_0^L p(t, \tilde{\omega}) \bar{l}(t, \omega, \tilde{\omega}) d\tilde{\omega},$$

where $\bar{l}(t, \omega, \tilde{\omega})$ is the land that the household at ω owns at $\tilde{\omega}$. The total value of wealth $v(t, \omega)$ owned by the household at ω is the sum of the two assets' values.

$$v(t, \omega) = \bar{k}(t, \omega) + a(t, \omega). \tag{9}$$

The household at ω collects the following rent from the land that the household owns.

$$\bar{r}(t, \omega) = \int_0^L R(t, \tilde{\omega}) \bar{l}(t, \omega, \tilde{\omega}) d\tilde{\omega}, \ \ 0 \le \omega \le L. \tag{10}$$

The total land rent of the economy is equal to the land rent that the population owns.

$$\int_0^L \bar{r}(t, \omega) d\omega = \int_0^L R(t, \omega) d\omega, \ \ 0 \le \omega \le L. \tag{11}$$

The household at ω has the following current income

$$y(t, \omega) = r^* \bar{k}(t, \omega) + w + \bar{r}(t, \omega), \ \ 0 \le \omega \le L, \tag{12}$$

from the interest payment rk, and the wage payment w, and the land rent income $\bar{r}$. We call $y(t, \omega)$ the current income in the sense that it comes from consumers' wages and current earnings from ownership of wealth. The total value of the wealth that a consumer at location ω can sell to purchase goods and to save is equal to $v(t, \omega)$. The disposable income $\hat{y}(t, \omega)$ is the sum of the current income and the total value of wealth .

$$\hat{y}(t, \omega) = y(t, \omega) + v(t, \omega). \tag{13}$$

2.7. The Budget

At each point in time, the household at location ω distributes the total available budget between housing $l(t, \omega)$, saving $s(t, \omega)$, consumption of industrial goods $c(t, \omega)$, and travelling, $c_T(\omega)$. The total expenditure is.

$$R(t, \omega) l(t, \omega) + c(t, \omega) + s(t, \omega) + c_T(t, \omega), \ \ 0 \le \omega \le L.$$

The disposable income equals the total expenditure, i.e.

$$R(t, \omega) l(t, \omega) + c(t, \omega) + s(t, \omega) + c_T(t, \omega) = \hat{y}(t, \omega), \ \ 0 \le \omega \le L. \tag{14}$$

Insert (11) and (10) in (12)

$$R(t, \omega) l(t, \omega) + c(t, \omega) + s(t, \omega) = \bar{y}(t, \omega), \tag{15}$$

where

$$\bar{y}(t, \omega) \equiv (1 + r^*) \bar{k}(t, \omega) + w + \bar{r}(t, \omega) + a(t, \omega) - c_T(t, \omega).$$

2.8. Utility, Amenity and Optimal Solution

We assume that utility level $U(t,\omega)$ of the household at location ω is dependent on $T(\omega)$, $l(t,\omega)$, $s(t,\omega)$, and $c(t,\omega)$ as follows.

$$U(t,\omega)=\theta(t,\omega)T^{\sigma_0}(\omega)c^{\xi_0}(t,\omega)l^{\eta_0}(t,\omega)s^{\lambda_0}(t,\omega),\ \ \sigma_0,\xi_0,\eta_0,\lambda_0>0, \tag{16}$$

in which σ_0, ξ_0, η_0, and λ_0 are a typical person's elasticity of utility with regard to leisure time, industrial goods, housing, and saving. We call σ_0, ξ_0, η_0, and λ_0 propensities to use leisure time, to consume goods, to consume housing, and to hold wealth, respectively. We specify the amenity $\theta(\omega,t)$, at ω as follows.

$$\theta(t,\omega)=\theta_1 n^{\mu}(t,\omega),\ \ \theta_1>0. \tag{17}$$

The function, $\theta(t,\omega)$, implies that the amenity level at location ω is related to the residential density at the location. Maximizing $U(t,\omega)$ subject to the budget constraint (8) yields.

$$l(t,\omega)=\frac{\eta\,\bar{y}(t,\omega)}{R(t,\omega)},\ \ c(t,\omega)=\xi\,\bar{y}(t,\omega),\ \ s(t,\omega)=\lambda\,\bar{y}(t,\omega), \tag{18}$$

where

$$\eta\equiv\rho\eta_0,\ \ \xi=\rho\xi_0,\ \ \lambda\equiv\rho\lambda_0,\ \ \frac{1}{\eta_0+\xi_0+\lambda_0}.$$

2.9. Equal Utility Level over the Residential Area

The assumption households get the same level of utility at any location at any point is represented by

$$U(t,\omega_1)=U(t,\omega_2),\ \ 0\le\omega_1,\ \ \omega_2\le L. \tag{19}$$

2.10. Wealth Accumulation

According to the definition of $s(t,\omega)$, the wealth accumulation of the household at location ω is given by

$$\dot{v}(t,\omega)=s(t,\omega)-v(t,\omega),\ \ 0\le\omega\le L. \tag{20}$$

2.11. The Land Market Equilibrium

According to the definition the total value of the national land $V(t)$ is

$$V(t)=\int_0^L p(t,\omega)d\omega. \tag{21}$$

The total value of land owned by the population is given by

$$V^*(t)=\int_0^L n(t,\omega)a(t,\omega)d\omega. \tag{22}$$

As the land is privately owned, the two values should equal

$$\int_0^L n(t,\omega)a(t,\omega)d\omega=\int_0^L p(t,\omega)d\omega. \tag{23}$$

2.12. The Equilibrium for Good Production and Consumption

The total consumption, $C(t)$, is given by

$$C(t)=\int_0^L n(t,\omega)c(t,\omega)d\omega. \tag{24}$$

We built the spatial growth model with residential location for a small-open economy.

3. The Spatial Dynamics

This section examines properties of the spatial model. The following lemma provides a computational procedure to plot the motion of the economic system.

3.1. Lemma

Assume $c_T = \bar{c}\, w(t)$, where $\bar{c}$ is a constant. The dynamics of wealth per household $v(t)$ is described by the following differential equation

$$\dot{v}(t) = \Omega(v(t)), \tag{25}$$

in which Ω is function of $v(t)$ defined in the appendix. For given $v(t)$, we uniquely determine all the other variables by the following procedure: w by (4) $\rightarrow$ K by (4) $\rightarrow$ F by (2) $\rightarrow$ $\bar{y}(t)$ by (A3) $\rightarrow$ $c(t)$ by (18) $\rightarrow$ $s(t)$ by (18) $\rightarrow$ $n(0)$ by (A17) $\rightarrow$ $n(\omega)$ by (A16) $\rightarrow$ $l(\omega)$ by (5) $\rightarrow$ $R(t, \omega)$ by (A5) $\rightarrow$ $p(t, \omega)$ by (6) $\rightarrow$ $U(t, \omega)$ by (A14) $\rightarrow$ $\tilde{v}(t) = \int_0^L n(\omega) v(\omega) d\omega$ $\rightarrow$ $\tilde{r}(t) = \int_0^L R(\omega) d\omega$ $\rightarrow$ $\bar{K}(t)$ by (A9) $\rightarrow$ $C(t)$ by (A12).

In the lemma $\tilde{v}(t)$ is the total wealth defined by

$$\tilde{v}(t) \equiv \int_0^L n(t, \omega) v(t, \omega) d\omega.$$

As it is difficult to analytically describe the spatial dynamics, we specify the parameters as follows

$$r^* = 0.07,\ \alpha = 0.45,\ A = 0.8,\ N = 50,\ L = 1,\ T_0 = 1,\ \delta_k = 0.03,\ \lambda_0 = 0.8,\ \sigma_0 = 0.2,$$
$$\xi_0 = 0.1,\ \eta_0 = 0.02,\ \bar{c} = 0.01,\ \theta_1 = 1,\ \mu = -0.05. \tag{26}$$

We specify the following initial conditions: $v(0) = 15.$ Under (26), we have

$$K = 513.5,\quad w = 1.26,\quad F = 114.1.$$

We plot the variables over time and space in in Figure 1.

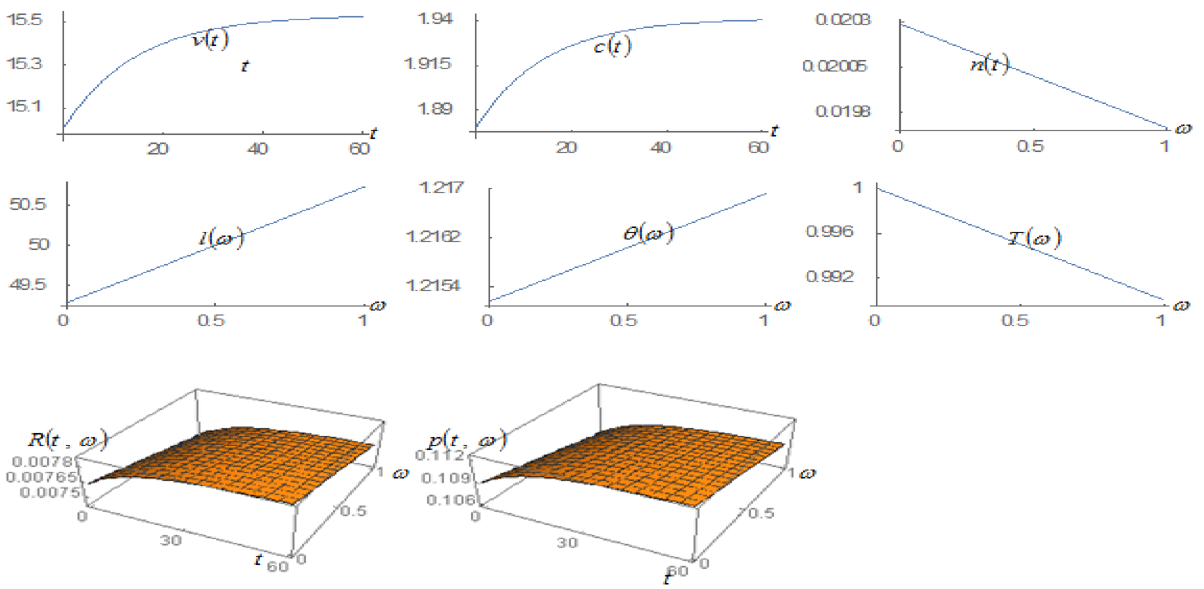

Figure-1. The Motion of the Economy over Space.

4. Comparative Dynamic Analysis

This section carries out comparative dynamic analysis with regards some parameters. We introduce a symbol $\overline{\Delta} x$ to stand for the change rate of the variable x in percentage due to changes in value of a parameter value.

4.1. An Increase in the Rate of Interest

We now allow the rate of interest to be increased as follows: $r^* : 0.07 \Rightarrow 0.075$. The change causes the total capital, wage rate and national output to follow as follows:

$$\bar{\Delta} K = -0.09, \quad \bar{\Delta} w = -0.04, \quad \bar{\Delta} F = -0.04.$$

The changes in the other variables are plotted in Figure 2. The total wealth per capita and consumption level per capita are increased. The lot size, amenity and leisure time are not affected. The land rent is increased and the land price is reduced.

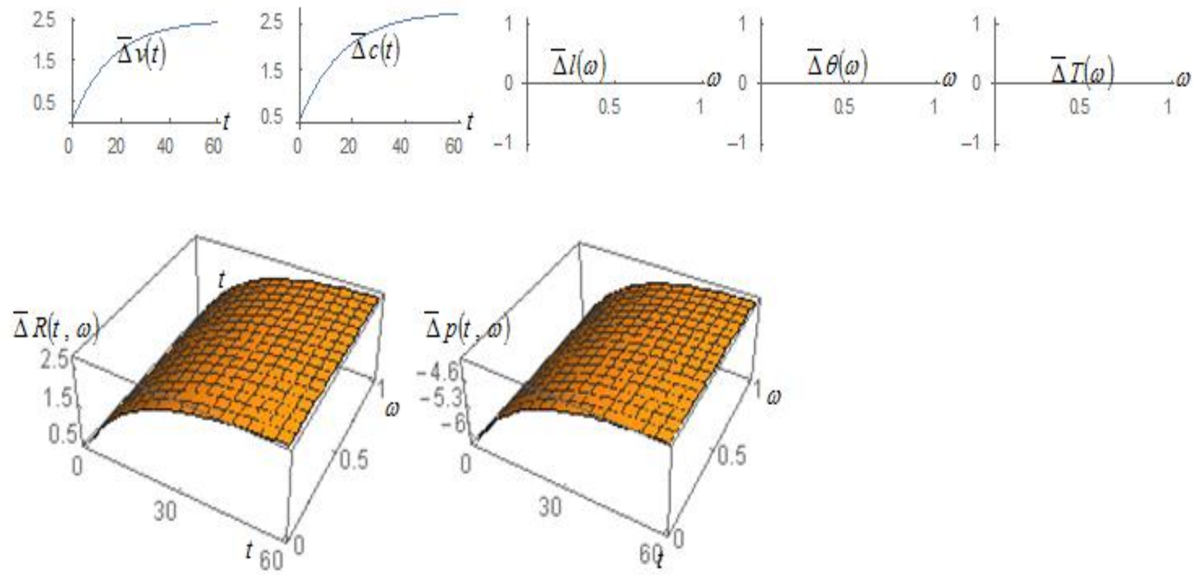

Figure-2. A Rise in the Rate of Interest.

4.2. The Total Factor Productivity Being Enhanced

We now examine the effects of an enhancement in the total factor productivity as follows: $A_0 : 0.8 \Rightarrow 0.81$. The change has no impact on the residential density, the lot size, the amenity, and leisure time. The total capital employed by the country, the wage rate and the national output are augmented

$$\bar{\Delta} K = \bar{\Delta} w = \bar{\Delta} F = 0.023.$$

The simulation results on the other variables are plotted in Figure 3. The total wealth and consumption level per capita are enhanced. The land value and rent are increased over time and space.

4.3. The Propensity to Save Being Increased

We now examine what happen to the economic system when the propensity to save is changed as follows: $\lambda_0 : 0.8 \Rightarrow 0.81$. There are no changes in the residential density, the lot size, the amenity, leisure time, and K, w, and F. The simulation results on the other variables are plotted in Figure 4. The total wealth and consumption level per capita are enhanced. The land value and rent are increased over time and space

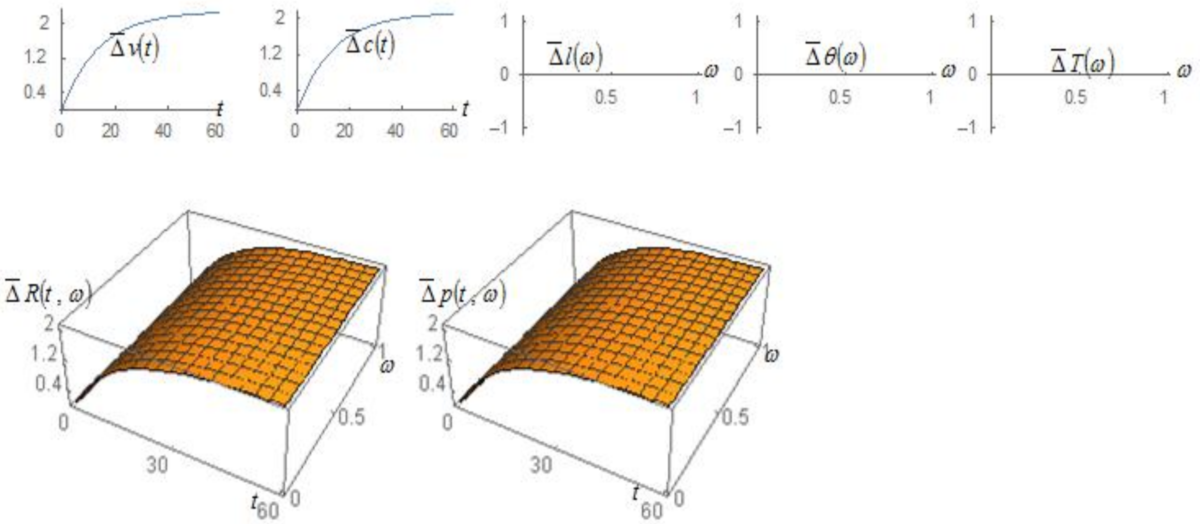

Figure-3. The Total Factor Productivity Being Enhanced.

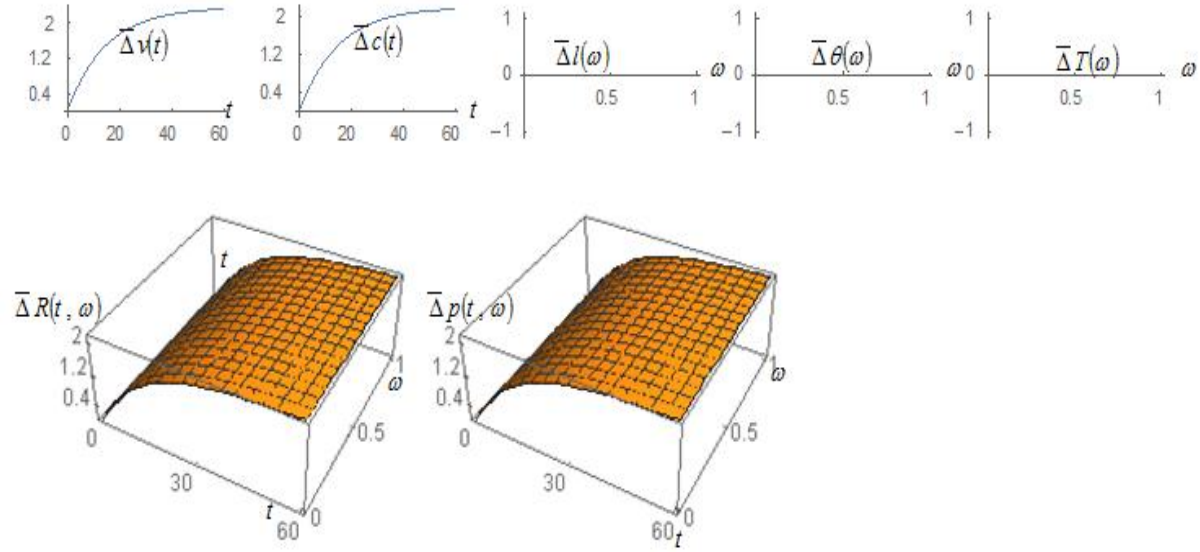

Figure-4. The Propensity to Save Being Increased.

5. Concluding Remarks

This study dealt with economic growth and residential distribution of a small-open economy. The unique contribution of the paper is that the model integrates the Solow growth and Alonso residential distribution models with endogenous land value. The economic system endogenously determines land value and rent with interactions between wealth accumulation, amenity, land, and transportation conditions. We simulated the motion of the spatial economy over time and space. We also carried out comparative dynamic analysis with regard to the rate of interest, the total productivity of the industrial sector, and preference on the spatial dynamics. Although the model is developed with microeconomic foundation and deals with complicated interactions among many variables over time and space, it is based on many strict assumptions. Many limitations of the model become apparent in the light of the sophistication of the literature of economic growth theory, regional science and urban economics.

References

Alonso, W. (1964). Location and land use. MA, Cambridge: Harvard University Press.

Solow, R. (1956). A contribution to the theory of growth. Quarterly Journal of Economics, 70(1): 65-94.

Obstfeld, M. & Rogoff, K. (1996). Foundations of international macroeconomics. Mass, Cambridge: MIT Press.

Zhang, W.B. (2010). Growth, economic structure, and residential distribution of a small city. International Review of Economics, 57(1): 47-78.

Zhang, W.B. (2014). Land value and rent dynamics in an integrated Walrasian general equilibrium and neoclassical growth theory. Annals of the Alexandru Ioan Cuza University of Iasi. Economic Sciences Section, 61(2): 235-58.

Zhang, W.B. (2018). Growth and residential location with endogenous land value in a small-open economy. Paper Presented at VIII FIMEF Financial Research Conference, Monterrey, Mexico, August 30 and 31, 2018.

Appendix

Insert the definition of $\bar{r}(\omega)$ in the definition of $\bar{y}$

$$\bar{y}(\omega) = (1 + r^*)\bar{k}(\omega) + w + \int_0^L R(\tilde{\omega})\bar{l}(\omega, \tilde{\omega})d\,\tilde{\omega} + \int_0^L p(\tilde{\omega})\bar{l}(\omega, \tilde{\omega})d\,\tilde{\omega} - c_T. \tag{A1}$$

Insert (6) in (A1)

$$\bar{y}(\omega) = (1 + r^*)\bar{k}(\omega) + w + (1 + r^*)\int_0^L p(\tilde{\omega})\bar{l}(\omega, \tilde{\omega})d\,\tilde{\omega} - c_T. \tag{A2}$$

Insert (9) and (10) in (A2)

$$\bar{y}(\omega) = (1 + r^*)v(\omega) + w - c_T. \tag{A3}$$

From (18) and (A3), we have

$$R(\omega)l(\omega) = (1 + r)\eta\, v(\omega) + \eta\, w - \eta\, c_T. \tag{A4}$$

Insert (5) in (A4)

$$R(\omega) = (1 + r^*)\eta\, n(\omega)v(\omega) + \eta\, n(\omega)w - \eta\, n(\omega)c_T. \tag{A5}$$

Integrate (A5) from 0 to L

$$\tilde{r} = (1 + r^*)\eta\tilde{v} + \eta\, wN - \eta\, c_T\, N, \tag{A6}$$

where we use (12) and

$$\tilde{r} \equiv \int_0^L R(\omega)d\,\omega, \;\; \tilde{v} \equiv \int_0^L n(\omega)v(\omega)d\,\omega.$$

From the definitions of v we have

$$\tilde{v} = \int_0^L \left(n(\omega)\bar{k}(\omega) + n(\omega)a(\omega)\right)d\,\omega = \bar{K} + V^*. \tag{A7}$$

where $\bar{K} \equiv \int_0^L n(\omega)\bar{k}(\omega)d\,\omega$. From (6) we have

$$\tilde{r} = rV^*. \tag{A8}$$

From (9) and (10) we have

$$\tilde{v} = \bar{K} + \frac{\tilde{r}}{r^*}. \tag{A9}$$

From (A6) and (A9) we solve

$$\tilde{v} = \hat{r}\left(r^*\bar{K} + \eta\, wN - \eta\, c_T\, N\right), \tag{A10}$$

where

$$\hat{r} = \frac{1}{r^* - (1 + r^*)\eta}.$$

From (18) and (A3), we have

$$c = (1 + r^*)\xi\, v + \xi\, w - \xi\, c_T, \tag{A11}$$

Multiplying the two sides of (A11) by n and then integrate the resulted equation from 0 to L

$$C = (1 + r^*)\xi\tilde{v} + \xi\, wN - \xi\, c_T\, N. \tag{A12}$$

Insert (18) and (A4) in (20)

$$\dot{v} = \Omega(v) \equiv -\bar{r}\, v + \lambda\, w - \lambda\, c_T, \tag{A13}$$

where $\bar{r} \equiv 1 - \lambda - \lambda r^*$. This is a first-order linear differential equation with constant coefficients. We denote its solution by $v(\omega, t)$. In the rest of the appendix we treat $v(\omega, t)$ as known functions of time. Insert (17), (18) and (4) in (16)

$$U(\omega) = \theta_0 n^{\mu-\eta_0}(\omega) T^{\sigma_0}(\omega) \bar{y}^{\xi_0+\lambda_0}(\omega). \tag{A14}$$

where $\theta_0 \equiv \theta_1 \lambda^{\lambda_0} \xi^{\xi_0}$. Insert (A14) in (19)

$$n^{\mu-\eta_0}(\omega) T^{\sigma_0}(\omega) \bar{y}^{\xi_0+\lambda_0}(\omega) = \hat{n}, \ \ 0 \le \omega_1, \ \ \omega_2 \le L, \tag{A15}$$

where

$$\hat{n}(t) = n^{\mu-\eta_0}(0) T^{\sigma_0}(0) \bar{y}^{\xi_0+\lambda_0}(0).$$

From (A15) we solve

$$n(\omega, t) = n(0, t) g(\omega, t), \ \ 0 \le \omega_1, \ \ \omega_2 \le L, \tag{A16}$$

where

$$g(\omega, t) \equiv \left(\frac{T^{\sigma_0}(0) \bar{y}^{\xi_0+\lambda_0}(0)}{T^{\sigma_0}(\omega) \bar{y}^{\xi_0+\lambda_0}(\omega)} \right)^{1/(\mu-\eta_0)}.$$

Insert (A16) in (1)

$$n(0, t) = N \left(\int_0^L g(\omega, t) d\omega \right)^{-1}. \tag{A17}$$

We can now determine all the variables over time and space by the procedure in the lemma.